Ejercicio

Calcular la integral de $f(x) = \frac{1}{x}$ entre x = 1 y x = 2; que esto es igual a ln 2 = 0.693147.

Se calculará el primer nivel en donde de realizará la regla del trapecio dependiendo de las particiones que indique cada sección. Se hará con 5 secciones:

A1:

$$h = \frac{x_1 - x_0}{2^0} = \frac{2-1}{1} = 1$$

$$\int_1^2 \frac{1}{x} dx = \frac{1}{2}\left(\frac{1}{1} + \frac{1}{2}\right) = 0.75$$

A2:

$$h = \frac{x_1 - x_0}{2^1} = \frac{2-1}{2} = 0.5$$

$$\int_1^{1.5} \frac{1}{x} dx = \frac{0.5}{2}\left(\frac{1}{1} + \frac{1}{1.5}\right) = 0.416666$$

$$\int_{1.5}^{2} \frac{1}{x} dx = \frac{0.5}{2}\left(\frac{1}{1.5} + \frac{1}{2}\right) = 0.291666$$

A2 = 0.416666 + 0.291666 = 0.708332

A3:

$$h = \frac{x_1 - x_0}{2^2} = \frac{2-1}{4} = 0.25$$

$$\int_{1}^{1.25} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1} + \frac{1}{1.25}\right) = 0.225$$

$$\int_{1.25}^{1.5} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.25} + \frac{1}{1.5}\right) = 0.183333$$

$$\int_{1.5}^{1.75} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.5} + \frac{1}{1.75}\right) = 0.154761$$

$$\int_{1.75}^{2} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.75} + \frac{1}{2}\right) = 0.133928$$

A3 = 0.225 + 0.183333 + 0.154761 + 0.133928 = 0.697022

A4:

$$h = \frac{x_1 - x_0}{2^3} = \frac{2-1}{8} = 0.125$$

$$\int_{1}^{1.125} \frac{1}{x}dx = \frac{0.125}{2}\left(\frac{1}{1} + \frac{1}{1.125}\right) = 0.118055$$

$$\int_{1.125}^{1.25} \frac{1}{x}dx = \frac{0.5}{2}\left(\frac{1}{1.125} + \frac{1}{1.25}\right) = 0.105555$$

$$\int_{1.25}^{1.375} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.25} + \frac{1}{1.375}\right) = 0.095454$$

$$\int_{1.375}^{1.5} \frac{1}{x}dx = \frac{0.5}{2}\left(\frac{1}{1.375} + \frac{1}{1.5}\right) = 0.087121$$

$$\int_{1.5}^{1.625} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.5} + \frac{1}{1.625}\right) = 0.080128$$

$$\int_{1.625}^{1.75} \frac{1}{x}dx = \frac{0.5}{2}\left(\frac{1}{1.25} + \frac{1}{1.5}\right) = 0.074175$$

$$\int_{1.75}^{1.875} \frac{1}{x}dx = \frac{0.25}{2}\left(\frac{1}{1.75} + \frac{1}{1.875}\right) = 0.069047$$

$$\int_{1.875}^{2} \frac{1}{x}dx = \frac{0.5}{2}\left(\frac{1}{1.75} + \frac{1}{2}\right) = 0.064583$$

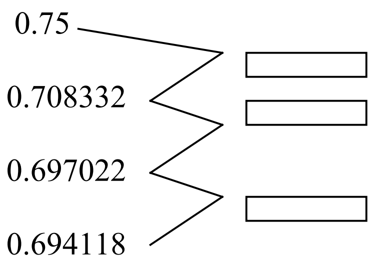
A4 = 0.118055 + 0.105555 + 0.095454 + 0.087121 + 0.080128 + 0.074175 + 0.069047 + 0.064583 = 0.694118

A5:

$$h = \frac{x_1 - x_0}{2^4} = \frac{2-1}{16} = 0.0625$$

A5: 0.060592 + 0.057127 + 0.054038 + 0.051265 + 0.048764 + 0.046495 + 0.044428 + 0.042537 + 0.040801 + 0.039201 + 0.037721 + 0.036350 + 0.035074 + 0.033885 + 0.032774 + 0.031734 = 0.692786

Ya teniendo las A procedemos a ocupar el método de Romberg.



y la formula de Romberg seria para este caso:

$$A_n = \frac{4^1 A_{n+1} - A_n}{4^1 - 1}$$

$$A_1 = \frac{4(0.708332) - 0.75}{3} = 0.694442$$

$$A_2 = \frac{4(0.697022) - 0.708332}{3} = 0.693252$$

$$A_3 = \frac{4(0.694118) - 0.697022}{3} = 0.69315$$

$$A_4 = \frac{4(0.692786) - 0.694118}{3} = 0.692342$$

Para la tercera columna será:

0.75		
	0.694442	
0.708332		[ ]
	0.692352	
0.697022		[ ]
	0.69315	
0.694118		

La formula de Romberg quedará así:

$$A_n^2 = \frac{4^2(A_{n+1}) - A_n}{4^2}$$

$$A_1^2 = \frac{4^2(0.692352) - 0.694442}{4^2 - 1} = 0.692212$$

$$A_2^2 = \frac{4^2(0.69315) - 0.692352}{4^2 - 1} = 0.693203$$

$$A_3^2 = \frac{4^2(0.692342) - 0.69315}{4^2 - 1} = 0.692288$$

Para la Cuarta columna será:

0.75			
	0.694442		
0.708332		0.692212	
	0.692352		[ ]
0.697022		0.693203	
	0.69315		
0.694118			

La formula de Romberg quedará así:

$$A_n^3 = \frac{4^3(A_{n+1}) - A_n}{4^3}$$

$$A_1^3 = \frac{4^3(0.693203) - 0.692212}{4^3 - 1} = 0.693218$$

$$A_2^3 = \frac{4^3(0.692288) - 0.693203}{4^3 - 1} = 0.692273$$

La quinta columna quedara así:

0.75			
	0.694442		
0.708332		0.692212	
	0.692352		0.693218
0.697022		0.693203	
	0.69315		
0.694118			

Y el resultado es:

$$A_n^4 = \frac{4^4(A_{n+1}) - A_n}{4^4}$$

$$A_1^4 = \frac{4^4(0.692273) - 0.693218}{4^4 - 1} = 0.692269$$

La integral total es: 0.692269