

Ejercicio

Aproxime el área bajo la curva de $\int_0^1 e^{x^2}$ usando el método de integración de Romberg, usando 1,2,4 trapecios o 1, $\frac{1}{2}$, $\frac{1}{4}$ (segmentos de longitud).

Mediante el método de trapecio compuesto se obtienen las primeras integrales.

$$I(h_1) = \frac{1-0}{2} [e^{0^2} + e^{1^2}] = .5(1 + 2.71828) = 1.85914$$

$$I(h_2) = \frac{1-0}{4} \left[e^{0^2} + e^{\left(\frac{1}{2}\right)^2} + e^{1^2} \right] = 0.25(1 + 2.56085 + 2.71828) = 1.571582$$

$$I(h_3) = \frac{1-0}{8} \left[e^{0^2} + 2 \left(e^{\left(\frac{1}{4}\right)^2} + e^{\left(\frac{1}{2}\right)^2} + e^{\left(\frac{3}{4}\right)^2} \right) + e^{1^2} \right]$$

$$I(h_3) = 0.125(1 + 2(e^{0.0625} + e^{.25} + e^{0.5625}) + e^1)$$

$$I(h_3) = 0.125(1 + 2(1.06449 + 1.28402 + 1.75505) + 2.71828)$$

$$I(h_3) = 0.125(1 + 2(4.10356) + 2.71828)$$

$$I(h_3) = 0.125(1 + 8.20712 + 2.71828)$$

$$I(h_3) = 0.125(11.9254)$$

$$I(h_3) = 1.490675$$

Mediante el uso de la siguiente formula se procede al calculo del área bajo la curva

$$I_k^{(m)} = \frac{4^m I_{k+1}^{(m-1)} - I_k^{(m-1)}}{4^m - 1}$$

Primera iteración

1.85914

1.571582

1.490675

$$\frac{4^1 * 1.571582 - 1.85914}{4^1 - 1}$$

$$\frac{4^1 * 1.490675 - 1.571582}{4^1 - 1}$$

$$\begin{array}{l} 1.85914 \\ 1.571582 \\ 1.490675 \end{array} \begin{array}{l} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} \begin{array}{l} \frac{4 \cdot 1.571582 - 1.85914}{4^1 - 1} \\ \frac{4 \cdot 1.490675 - 1.571582}{4^1 - 1} \end{array}$$

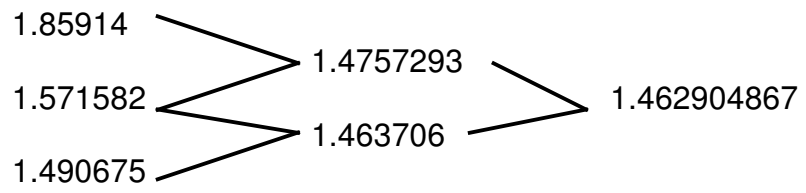
$$\begin{array}{l} 1.85914 \\ 1.571582 \\ 1.490675 \end{array} \begin{array}{l} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} \begin{array}{l} \frac{6.286328 - 1.85914}{3} \\ \frac{5.9627 - 1.571582}{3} \end{array}$$

$$\begin{array}{l} 1.85914 \\ 1.571582 \\ 1.490675 \end{array} \begin{array}{l} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} \begin{array}{l} \frac{4.427188}{3} \\ \frac{4.391118}{3} \end{array}$$

$$\begin{array}{l} 1.85914 \\ 1.571582 \\ 1.490675 \end{array} \begin{array}{l} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} \begin{array}{l} 1.4757293 \\ 1.463706 \end{array}$$

Segunda iteración

$$\begin{array}{l} 1.85914 \\ 1.571582 \\ 1.490675 \end{array} \begin{array}{l} \nearrow \\ \searrow \\ \nearrow \\ \searrow \\ \nearrow \\ \searrow \end{array} \begin{array}{l} 1.4757293 \\ 1.463706 \end{array} \begin{array}{l} \nearrow \\ \searrow \end{array} \frac{4 \cdot 1.463706 - 1.4757293}{4^2 - 1}$$



$$\int_0^1 e^{-x^2} \approx 1.462904867$$

