

Metodo de Runge - Kutta Cuarto orden

Utilice el método de Runge Kutta para calcular la solución en 5 intervalos (i) en $y(0.5)=?$ para la ecuación diferencial:

$$dy/dx=1-x+4y \quad \text{con} \quad y(0)=1$$

$$\begin{aligned} k_1 &= f(x_i, y_i) \\ k_2 &= f(x_i + (h/2), y_i + (hk_1/2)) \\ k_3 &= f(x_i + (h/2), y_i + (hk_2/2)) \\ k_4 &= f(x_i + h, y_i + hk_3) \\ y_{i+1}(x_{i+1}) &= y_i + [(h/6)(k_1 + 2k_2 + 2k_3 + k_4)] \\ h &= (x_b - x_a)/i \end{aligned}$$

$$\begin{aligned} h &= (0.5 - 0)/5 = 0.1 \\ x_0 &= 0, \quad x_1 = 0.1, \quad x_2 = 0.2, \quad x_3 = 0.3, \quad x_4 = 0.4, \quad x_5 = 0.5 \end{aligned}$$

$$\begin{aligned} i &= 0, \quad x_0 = 0, \quad y_0 = 1 \\ k_1 &= f(0, 1) = 5 \\ k_2 &= f(0 + (0.1/2), 1 + ((0.1 * 5)/2)) = f(0.05, 1.25) = 5.95 \\ k_3 &= f(0 + (0.1/2), 1 + ((0.1 * 5.95)/2)) = f(0.05, 1.2975) = 6.14 \\ k_4 &= f(0 + 0.1, 1 + (0.1 * 6.14)) = f(0.1, 1.614) = 7.356 \\ y_1(x_1) &= 1 + [(0.1/6)(5 + 2 * 5.95 + 2 * 6.14 + 7.356)] \\ y_1(0.1) &= 1.60893 \end{aligned}$$

$$\begin{aligned} i &= 1, \quad x_1 = 0.1 \\ k_1 &= f(0.1, y_1) = 7.33573 \\ k_2 &= f(0.1 + (0.1/2), y_1 + ((0.1 * k_1)/2)) = f(0.15, 1.97572) = 8.75288 \\ k_3 &= f(0.1 + (0.1/2), y_1 + ((0.1 * k_2)/2)) = f(0.15, 2.04658) = 9.03631 \\ k_4 &= f(0.1 + 0.1, y_1 + (0.1 * k_3)) = f(0.2, 2.51256) = 10.8503 \\ y_2(x_2) &= y_1 + [(0.1/6)(k_1 + 2 * k_2 + 2 * k_3 + k_4)] \\ y_2(0.2) &= 2.50501 \end{aligned}$$

$$\begin{aligned} i &= 2, \quad x_2 = 0.2 \\ k_1 &= f(0.2, y_2) = 10.82 \\ k_2 &= f(0.2 + (0.1/2), y_2 + ((0.1 * k_1)/2)) = f(0.25, 3.04601) = 12.934 \\ k_3 &= f(0.2 + (0.1/2), y_2 + ((0.1 * k_2)/2)) = f(0.25, 3.15171) = 13.3568 \\ k_4 &= f(0.2 + 0.1, y_2 + (0.1 * k_3)) = f(0.3, 3.84069) = 16.0628 \\ y_3(x_3) &= y_2 + [(0.1/6)(k_1 + 2 * k_2 + 2 * k_3 + k_4)] \\ y_3(0.3) &= 3.82941 \end{aligned}$$

$$\begin{aligned} i &= 3, \quad x_3 = 0.3 \\ k_1 &= f(0.3, y_3) = 16.0177 \\ k_2 &= f(0.3 + (0.1/2), y_3 + ((0.1 * k_1)/2)) = f(0.35, 4.6303) = 19.1712 \\ k_3 &= f(0.3 + (0.1/2), y_3 + ((0.1 * k_2)/2)) = f(0.35, 4.78797) = 19.8019 \\ k_4 &= f(0.3 + 0.1, y_3 + (0.1 * k_3)) = f(0.4, 5.8096) = 23.8384 \\ y_4(x_4) &= y_3 + [(0.1/6)(k_1 + 2 * k_2 + 2 * k_3 + k_4)] \\ y_4(0.4) &= 5.79279 \end{aligned}$$

$$i=4, \quad x_4=0.4$$

$$k_1=f(0.4,y_4)=23.7711$$

$$k_2=f(0.4+(0.1/2),y_4+(0.1*k_1)/2)=f(0.45,6.98134)=28.4754$$

$$k_3=f(0.4+(0.1/2),y_4+(0.1*k_2)/2)=f(0.45,7.21655)=29.4162$$

$$k_4=f(0.4+0.1,y_4+(0.1*k_3))=f(0.5,8.73441)=35.4376$$

$$y_5(x_5)=y_4+[(0.1/6)(k_1+2*k_2+2*k_3+k_4)]$$

$$y_5(0.5)=8.70932$$