

Método de Runge - Kutta Cuarto orden

Utilice el método de Runge Kutta para calcular la solución en 2 intervalos (i) de $x_a=0.1$ a $x_b=0.3$ para la ecuación diferencial:

$$dy/dx=x^2+y^2 \quad \text{con} \\ y(0)=1$$

$$\begin{aligned} k_1 &= f(x_i, y_i) \\ k_2 &= f(x_i + (h/2), y_i + (hk_1/2)) \\ k_3 &= f(x_i + (h/2), y_i + (hk_2/2)) \\ k_4 &= f(x_i + h, y_i + hk_3) \\ y_{i+1}(x_{i+1}) &= y_i + [(h/6)(k_1 + 2k_2 + 2k_3 + k_4)] \\ h &= (x_b - x_a)/i \end{aligned}$$

$$\begin{aligned} h &= (0.3 - 0.1)/2 = 0.1 \\ x_0 &= 0, \quad x_1 = 0.1, \quad x_2 = 0.2, \quad x_3 = 0.3 \end{aligned}$$

$$\begin{aligned} i &= 0, \quad x_0 = 0, \quad y_0 = 1 \\ k_1 &= f(0, 1) = 1 \\ k_2 &= f(0 + (0.1/2), 1 + ((0.1 * 1)/2)) = f(0.05, 1.05) = 1.105 \\ k_3 &= f(0 + (0.1/2), 1 + ((0.1 * 1.05)/2)) = f(0.05, 1.0525) = 1.116052563 \\ k_4 &= f(0 + 0.1, 1 + (0.1 * 1.116052563)) = f(0.1, 1.111605256) = 1.245666246 \\ y_1(x_1) &= 1 + [(0.1/6)(1 + 2 * 1.105 + 2 * 1.116052563 + 1.245666246)] \\ y_1(0.1) &= 1.111462856 \end{aligned}$$

$$\begin{aligned} i &= 1, \quad x_1 = 0.1 \\ k_1 &= f(0.1, y_1) = 1.24534968 \\ k_2 &= f(0.1 + (0.1/2), y_1 + ((0.1 * k_1)/2)) = f(0.15, 1.17373034) = 1.400142911 \\ k_3 &= f(0.1 + (0.1/2), y_1 + ((0.1 * k_2)/2)) = f(0.15, 1.181470002) = 1.418371365 \\ k_4 &= f(0.1 + 0.1, y_1 + (0.1 * k_3)) = f(0.2, 1.253299992) = 1.610760871 \\ y_2(x_2) &= y_1 + [(0.1/6)(k_1 + 2 * k_2 + 2 * k_3 + k_4)] \\ y_2(0.2) &= 1.253015174 \end{aligned}$$

$$\begin{aligned} i &= 2, \quad x_2 = 0.2 \\ k_1 &= f(0.2, y_2) = 1.610047026 \\ k_2 &= f(0.2 + (0.1/2), y_2 + ((0.1 * k_1)/2)) = f(0.25, 1.333517525) = 1.84076899 \\ k_3 &= f(0.2 + (0.1/2), y_2 + ((0.1 * k_2)/2)) = f(0.25, 1.345053624) = 1.87166925 \\ k_4 &= f(0.2 + 0.1, y_2 + (0.1 * k_3)) = f(0.3, 1.440182099) = 2.164124478 \\ y_3(x_3) &= y_2 + [(0.1/6)(k_1 + 2 * k_2 + 2 * k_3 + k_4)] \\ y_3(0.3) &= 1.439665974 \end{aligned}$$