

EJEMPLO : Resolver por Runge Kutta 4to Orden, la ecuación diferencial $y' - 2xy = x$, con $h=0.1$ en $0.5 < x < 1.0$ con condiciones iniciales $y(0.5)=1.0$

Solución:

Obtenemos los datos:

$$y' = x + 2xy$$

$$h = 0.1$$

$$y_0 = 1$$

$$x_0 = 0.5$$

Primera iteración(para i=0):

De la formula $Y_{i+1} = Y_i + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4]$

Calculamos a K_1 :

$$\begin{aligned} K_1 &= f(X_i, Y_i) \\ &= 0.5 + 2(0.5)(1) \\ &= 1.5 \end{aligned}$$

Calculamos a K_2 :

$$\begin{aligned} K_2 &= f\left(X_i + \frac{h}{2}, Y_i + \frac{h}{2} K_1\right) \\ &= f(0.55, 1.0075) \\ &= 0.55 + 2(0.55)(1.0075) \\ &= 1.65825 \end{aligned}$$

Calculamos a K_3 :

$$\begin{aligned} K_3 &= f\left(X_i + \frac{h}{2}, Y_i + \frac{h}{2} K_2\right) \\ &= f\left(0.55, 1 + \frac{0.1}{2} \cdot 1.65825\right) \\ &= f(0.55, 1.0829) \\ &= 0.55 + 2(0.55)(1.0829) \\ &= 1.7412 \end{aligned}$$

Calculamos a K_4 :

$$\begin{aligned} K_4 &= f(X_i + h, Y_i + h K_3) \\ &= f(0.5 + 0.1, (1 + 0.1(1.7412))) \\ &= 0.6 + 2(0.6)(1.7412) \\ &= 2.0089 \end{aligned}$$

Ahora sustituimos en la formula:

$$Y_{i+1} = Y_i + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$Y_1 = 1 + \frac{0.1}{6} [1.5 + 2(1.65825) + 2(1.7412) + 2.0089]$$

$$Y_1 = 1 + 0.016666[10.3078]$$

$$Y_1 = 1.17717$$

Obtenemos los nuevos datos:

$$y' = x + 2xy$$

$$h = 0.1$$

$$y_0 = 1.1717$$

$$x_0 = 0.6$$

Segunda iteración(para i=1):

$$\text{De la formula } Y_{i+1} = Y_i + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

Calculamos a K₁:

$$\begin{aligned} K_1 &= f(X_i, Y_i) \\ &= 0.6 + 2(0.6)(1.1717) \\ &= 2.006 \end{aligned}$$

Calculamos a K₂:

$$\begin{aligned} K_2 &= f\left(X_i + \frac{h}{2}, Y_i + \frac{h}{2} K_1\right) \\ &= f(0.65, 1.2772) \\ &= 0.65 + 2(0.65)(1.2772) \\ &= 2.3036 \end{aligned}$$

Calculamos a K₃:

$$\begin{aligned} K_3 &= f\left(X_i + \frac{h}{2}, Y_i + \frac{h}{2} K_2\right) \\ &= f\left(0.65, 1 + \frac{0.1}{2} 2.3036\right) \\ &= f(0.65, 1.2868) \\ &= 0.65 + 2(0.65)(1.2868) \\ &= 2.3229 \end{aligned}$$

Calculamos a K_4 :

$$\begin{aligned}K_4 &= f [(X_i+h), (Y_i + h K_1)] \\ &= f [(0.6+0.1), (1 + 0.1(2.3229))] \\ &= 0.7+2(0.7)(1.23229) \\ &= 2.4252\end{aligned}$$

Ahora sustituimos en la formula:

$$Y_{i+1} = Y_i + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$Y_2 = 1.1717 + \frac{0.1}{6} [2.006 + 4.6072 + 4.6458 + 2.4252]$$

$$Y_2 = 1.1717 + 0.016666 [13.6842]$$

$$Y_2 = 1.3997$$

Y asi sucesivamente vamos calculando las Y_i 's en este caso calculamos hasta " Y_6 ".