

Ejercicio

Resuelva el siguiente ejercicio por el método de Euler:

$$\text{PVI} \begin{cases} \frac{dy}{dx} = -2x - y \\ y(0) = -1 \\ y(2) = ? \end{cases}$$

Datos:

$$X_{\text{inicial}} = 0$$

$$X_{\text{final}} = 2$$

$$Y_{\text{inicial}} = -1$$

$$\text{Altura}(h) = 0.1$$

Formulas:

$$h = \frac{X_{\text{fin}} - X_{\text{in}}}{n(\text{intervalos})}$$

$$y = Y_{\text{in}} + h(f(X_{\text{in}}, Y_{\text{in}}))$$

Entonces nuestro numero de intervalos esta dado por:

$$h = \frac{X_{\text{fin}} - X_{\text{in}}}{n(\text{intervalos})} \quad n(\text{intervalos}) = \frac{X_{\text{fin}} - X_{\text{in}}}{h} = \frac{2 - 0}{0.1} = 20$$

Al utilizar 20 intervalos se tiene:

$X_1 = 0 + 0.1 = 0.1$	$X_6 = 0.5 + 0.1 = 0.6$	$X_{11} = 1 + 0.1 = 1.1$	$X_{16} = 1.5 + 0.1 = 1.6$
$X_2 = 0.1 + 0.1 = 0.2$	$X_7 = 0.6 + 0.1 = 0.7$	$X_{12} = 1.1 + 0.1 = 1.2$	$X_{17} = 1.6 + 0.1 = 1.7$
$X_3 = 0.2 + 0.1 = 0.3$	$X_8 = 0.7 + 0.1 = 0.8$	$X_{13} = 1.2 + 0.1 = 1.3$	$X_{18} = 1.7 + 0.1 = 1.8$
$X_4 = 0.3 + 0.1 = 0.4$	$X_9 = 0.8 + 0.1 = 0.9$	$X_{14} = 1.3 + 0.1 = 1.4$	$X_{19} = 1.8 + 0.1 = 1.9$
$X_5 = 0.4 + 0.1 = 0.5$	$X_{10} = 0.9 + 0.1 = 1$	$X_{15} = 1.4 + 0.1 = 1.5$	$X_{20} = 1.9 + 0.1 = 2$

ITERACIONES: $y = Y_{\text{in}} + h(f(X_{\text{in}}, Y_{\text{in}}))$

$$y_1 = -1 + 0.1(-2(0) - (-1)) = -0.9$$

$$Y_2 = -0.9 + 0.1(-2(0.1) - (-0.9)) = -0.83$$

$$Y_3 = -0.83 + 0.1(-2(0.2) - (-0.83)) = -0.787$$

$$Y_4 = -0.787 + 0.1(-2(0.3) - (-0.787)) = -0.7683$$

$$Y_5 = -0.7683 + 0.1(-2(0.4) - (-0.7683)) = -0.77147$$

$$Y_6 = -0.77147 + 0.1(-2(0.5) - (-0.77147)) = -0.794323$$

$$Y_7 = -0.794323 + 0.1(-2(0.6) - (-0.794323)) = -0.834891$$

$$Y_8 = -0.834891 + 0.1(-2(0.7) - (-0.834891)) = -0.891402$$

$$Y_9 = -0.891402 + 0.1(-2(0.8) - (-0.891402)) = -0.962261$$

$$Y_{10} = -0.962261 + 0.1(-2(0.9) - (-0.962261)) = -1.046035$$

$$Y_{11} = -1.046035 + 0.1(-2(1) - (-1.046035)) = -1.141432$$

$$Y_{12} = -1.141432 + 0.1(-2(1.1) - (-1.141432)) = -1.247289$$

$$Y_{13} = -1.247289 + 0.1(-2(1.2) - (-1.247289)) = -1.36256$$

$$Y_{14} = -1.36256 + 0.1(-2(1.3) - (-1.36256)) = -1.486304$$

$$Y_{15} = -1.486304 + 0.1(-2(1.4) - (-1.486304)) = -1.617674$$

$$Y_{16} = -1.617674 + 0.1(-2(1.5) - (-1.617674)) = -1.755906$$

$$Y_{17} = -1.755906 + 0.1(-2(1.6) - (-1.755906)) = -1.900316$$

$$Y_{18} = -1.900316 + 0.1(-2(1.7) - (-1.900316)) = -2.050284$$

$$Y_{19} = -2.050284 + 0.1(-2(1.8) - (-2.050284)) = -2.205256$$

$$Y_{20} = -2.205256 + 0.1(-2(1.9) - (-2.205256)) = -2.36473$$

SOLUCIÓN A LA ECUACIÓN DIFERENCIAL ORDINARIA:

Obteniendo finalmente:

$$Y(2) = -2.36473$$

Resolviendo analíticamente: $\frac{dy}{dx} = -2x - y$

$$Y(2) = -2.405839$$

Entonces:

$$-2.405839 \approx -2.36473$$